Lecture 8: Signal Transforms (Cont.)

Introduction

Decimation

If a sequence $x[k]$ is compressed by a factor c, some data samples of $x[k]$ are lost. For example, if we decimate $x[k]$ by 2, the decimated function $y[k] = x[2k]$ retains only the alternate samples given by $x[0]$, $x[2]$, $x[4]$, and so on.
Compression (referred to as decimation for DT sequences) is, therefore, an irreversible process in the DT domain as the original sequence $x[k]$ cannot be recovered precisely from the decimated sequence $y[k]$.

Interpolation

In the DT domain, expansion (also referred to as interpolation) is defined as follows:

$$x^{(m)}[k] = \begin{cases} x\left[\frac{k}{m}\right] & \text{if } k \text{ is a multiple of integer } m \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

The interpolated sequence $x^{(m)}[k]$ inserts $(m-1)$ zeros in between adjacent samples of the DT sequence $x[k]$. Interpolation of the DT sequence $x[k]$is a reversible process as the original sequence $x[k]$ can be recovered from $x^{(m)}[k]$

Example: Decimation & interpolation

Consider the DT sequence $x[k]$ plotted in Fig. 1(a). Calculate and sketch $p[k] = x[2k]$ and $q[k] = x[k/2]$.

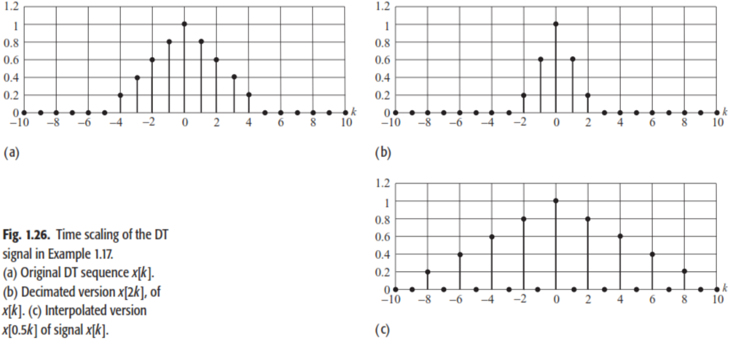

Fig. 1.26. Time scaling of the DT signal in Example 1.17. (a) Original DT sequence $x[k]$. (b) Decimated version $x[2k]$, of $x[k]$. (c) Interpolated version $x[0.5k]$ of signal $x[k]$.

Figure 1: Decimation & interpolation

Example: Decimation & interpolation

Solution:

Since $x[k]$ is non-zero for$-5 \leq k \leq 5$, the non-zero values of the decimated sequence $p[k] = x[2k]$ lie in the range $-3 \leq k \leq 3$. The non-zero values of $p[k]$ are shown in Table 1.2 (see textbook p.42). The waveform for p[k] is plotted in Fig. 1(b). The waveform for the decimated sequence $p[k]$ can be obtained by directly compressing the waveform for $x[k]$ by a factor of 2 about the x-axis.

While performing the compression, the value of $x[k]$at$k = 0$ is retained in $p[k]$. On both sides of the $k = 0$ sample, every second sample of $x[k]$ is retained in $p[k]$.

Example: Decimation & interpolation

To determine $q[k] = x[k/2]$, we first determine the range over which $x[k/2]$ is non-zero. The non-zero values of $q[k] = x[k/2]$ lie in the range $-10 \leq k \leq 10$ and are shown in Table 1.3(see textbook p.42). The waveform for $q[k]$ is plotted in Fig. 1(c). The waveform for the decimated sequence $q[k]$ can be obtained by directly expanding the waveform for $x[k]$ by a factor of 2 about the x-axis. During expansion, the value of $x[k]$at$k = 0$ is retained in $q[k]$. The even-numbered samples, where k is a multiple of 2, of q[k] equal $x[k/2]$. The odd-numbered samples in $q[k]$ are set to zero.

Linear interpolation

While determining the interpolated sequence $x[mk]$, Eq. (1) inserts $(m-1)$ zeros in between adjacent samples of the DT sequence $x[k]$, where$x[k]$ is not defined. Instead of inserting zeros, we can possibly interpolate the undefined values from the neighboring samples where $x[k]$ is defined. Using linear interpolation, an interpolated sequence can be obtained using the following equation:

$$x^{(m)}[k] = \begin{cases} x\left[\frac{k}{m}\right] & \text{if } k \text{ is a multiple of integer } m \\ (1-\alpha)x\left[\lfloor\frac{k}{m}\rfloor\right] + \alpha x\left[\lceil\frac{k}{m}\rceil\right] & \text{otherwise} \end{cases} \tag{2}$$

Where $\lfloor\frac{k}{m}\rfloor$ denotes the nearest integer less than or equal to(k/m) ("floor" function), $\lceil\frac{k}{m}\rceil$ denotes the nearest integer greater than or equal to (k/m) ("ceil" function), and $\alpha = (kmodm)/m$. Note that mod is the modulo operator that calculates the remainder of the division k/m. We will use Eq. (1);

Combined operations

An arbitrary linear operation that combines the three transformations is expressed as $x(\alpha t + \beta)$, where α is the time-scaling factor and β is the time-shifting factor. If α is negative, the signal is inverted along with the time-scaling and time-shifting operations. By expressing the transformed signal as

$$x(\alpha t + \beta) = x\left(\alpha\left(t + \frac{\beta}{\alpha}\right)\right) \quad (3)$$

(i)Scale the signal $x(t)$ by$|\alpha|$. The resulting waveform represents $x(|\alpha|t)$.
(ii) If α is negative, invert the scaled signal $x(|\alpha|t)$ with respect to the $t = 0$ axis. This step produces the waveform for $x(\alpha t)$.
(iii) Shift the waveform for $x(\alpha t)$ obtained in step (ii) by $|\beta/\alpha|$ time units. Shift towards the right-hand side if β/α is negative. Otherwise, shift towards the left-hand side if β/α is positive. The waveform resulting from this step represents $x(\alpha t + \beta)$, which is the required transformation.

Example:Combined Transform

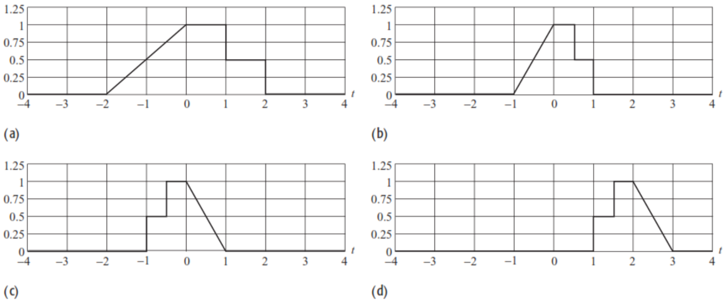

Figure 2: Combined transform

Determine $x(4-2t)$, where the waveform for the CT signal $x(t)$ is plotted above.

Example:Combined Transform

Solution:
Express $x(4-2t)$ =$x(-2[t-2])$ and follow steps (i)–(iii) as outlined below.

(i) Compress $x(t)$ by a factor of 2 to obtain $x(2t)$. The resulting waveform is shown in Fig. 2 (b).

(ii) Time-reverse $x(2t)$ to obtain $x(-2t)$. The waveform for $x(-2t)$ is shown in Fig. 2(c).

(iii) Shift $x(-2t)$ towards the right-hand side by two time units to obtain $x(-2[t-2]) = x(4-2t)$. The waveform for $x(4-2t)$ is plotted in Fig. 2(d).

Example:DT Combined Transform

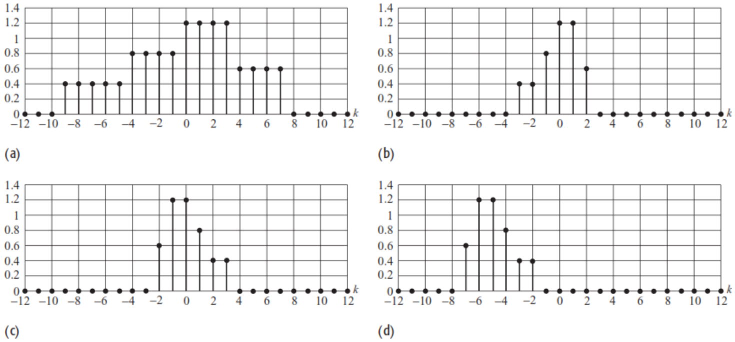

Figure 3: Combined transform

Sketch the waveform for $x[-15-3k]$ for the DT sequence $x[k]$ plotted in Fig. 3(a).

Example:DT Combined Transform

Solution:

Express $x[-15-3k]=x[-3(k+5)]$ and follow steps (i)–(iii) as outlined below.

(i) Compress $x[k]$ by a factor of 3 to obtain $x[3k]$. The resulting waveform is shown in Fig. 3 (b).

(ii) Time-reverse $x[3k]$ to obtain $x[-3k]$. The waveform for $x[-3k]$ is shown in Fig. 3(c).

(iii) Shift $x[-3k]$ towards the left-hand side by five time units to obtain $x[-3(k+5)]=x[-15-3k]$. The waveform for $x[-15-3k]$ is plotted in Fig. 3(d).