Lecture 10: System Classifications

Introduction

1. Time-varying and time-invariant systems
2. Systems with and without memory
3. Causal and non-causal systems
4. Invertible and non-invertible systems
5. Stable and unstable systems
6. System interconnections
7. Cascaded configuration
8. Parallel configuration
9. Feedback configuration

Time-varying and time-invariant systems

A system is said to be time-invariant (TI) if a time delay or time advance of the input signal leads to an identical time-shift in the output signal. In other words, except for a time-shift in the output, a TI system responds exactly the same way no matter when the input signal is applied. We now define a TI system formally.

Time-varying and time-invariant systems

$$\begin{aligned} \text{CT system} \quad & x(t) \to y(t) \\ \text{DT system} \quad & x[k] \to y[k] \end{aligned} \tag{1}$$

System is called time invariant if

$$\begin{aligned} \text{CT system} \quad & x(t-t_0) \to y(t-t_0) \\ \text{DT system} \quad & x[k-k_0] \to y[k-k_0] \end{aligned} \tag{2}$$

Systems with and without memory

A CT system is said to be without memory (memoryless or instantaneous) if its output $y(t)$ at time $t = t_0$ depends only on the values of the applied input $x(t)$ at the same time $t = t_0$. On the other hand, if the response of a system at $t = t_0$ depends on the values of the input$x(t)$ in the past or in the future of time $t = t_0$, it is called a dynamic system, or a system with memory. Likewise, a DT system is said to be memoryless if its output $y[k]$ at instant $k = k_0$ depends only on the value of its input $x[k]$ at the same instant $k = k_0$. Otherwise, the DT system is said to have memory.

Systems with and without memory

Table 2.1. Examples of CT and DT systems with and without memory

Continuous-time		Discrete-time	
Memoryless systems	Systems with memory	Memoryless systems	Systems with memory
$y(t) = 3x(t) + 5$	$y(t) = x(t-5)$	$y[k] = 3x[k] + 7$	$y[k] = x[k-5]$
$y(t) = \sin\{x(t)\} + 5$	$y(t) = x(t+2)$	$y[k] = \sin(x[k]) + 3$	$y[k] = x[k+3]$
$y(t) = e^{x(t)}$	$y(t) = x(2t)$	$y[k] = e^{x[k]}$	$y[k] = x[2k]$
$y(t) = x^2(t)$	$y(t) = x(t/2)$	$y[k] = x^2[k]$	$y[k] = x[k/2]$

Figure 1: Systems with and without memory

Causal and non-causal systems

A CT system is causal if the output at time t_0 depends only on the input $x(t)$ for $t \leq t_0$. Likewise, a DT system is causal if the output at time instant k_0 depends only on the input $x[k]$ for $k \leq k_0$. A system that violates the causality condition is called a non-causal (or anticipative) system. Note that all memoryless systems are causal systems because the output at any time instant depends only on the input at that time instant. Systems with memory can either be causal or non-causal.

Causal and non-causal systems

Example 2.7

(i) CT time-delay system $y(t) = x(t-2) \Rightarrow$ causal system;

(ii) CT time-forward system $y(t) = x(t+2) \Rightarrow$ non-causal system;

(iii) DT time-delay system $y[k] = x[k-2] \Rightarrow$ causal system;

(iv) DT time-advance system $y[k] = x[k+2] \Rightarrow$ non-causal system;

(v) DT linear system $y[k] = x[k-2] + x[k+10] \Rightarrow$ non-causal system.

Figure 2: Causal and non-causal systems

Invertible and non-invertible systems

A CT system is invertible if the input signal $x(t)$ can be uniquely determined from the output $y(t)$ produced in response to $x(t)$ for all time $t \in (-\infty, +\infty)$. Similarly, a DT system is called invertible if, given an arbitrary output response $y[k]$ of the system for $k \in (-\infty, +\infty)$, the corresponding input signal $x[k]$ can be uniquely determined for all time $k \in (-\infty, +\infty)$. To be invertible, two different inputs cannot produce the same output since, in such cases, the input signal cannot be uniquely determined from the output signal.

A direct consequence of the invertibility property is the determination of a second system that restores the original input. A system is said to be invertible if the input to the system can be recovered by applying the output of the original system as input to a second system. The second system is called the inverse of the original system.

Invertible and non-invertible systems

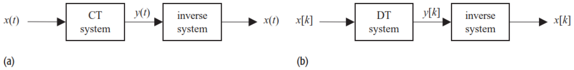

Figure 3: Invertible systems

Stable and unstable systems

We define the bounded property for a signal. A CT signal $x(t)$ or a DT signal $x[k]$ is said to be bounded in magnitude if

$$\begin{aligned} \text{CT system} \quad |x(t)| \leq B_x < \infty \text{ for} t \in (-\infty, \infty) \\ \text{DT system} |x[k]| \leq B_x < \infty \text{ for} k \in (-\infty, \infty) \end{aligned} \tag{3}$$

where Bx is a finite number. Next, we define the stability criteria for CT and DT systems.

Stable and unstable systems

A system is referred to as bounded-input, bounded-output (BIBO) stable if an arbitrary bounded-input signal always produces a bounded-output signal. In other words, if an input signal $x(t)$ for CT systems, or $x[k]$ for DT systems, is applied to a stable CT or DT system, it is always possible to find an finite number $B_y < \infty$ such that

$$\begin{aligned} &\text{CT system} \qquad |y(t)| \leq B_y < \infty \text{ for } t \in (-\infty, \infty) \\ &\text{DT system} \qquad |y[k]| \leq B_y < \infty \text{ for } k \in (-\infty, \infty) \end{aligned} \tag{4}$$

System interconnections

Two or more systems can be connected in different ways. Question is if we know input(or inputs), how determine output of combined system.

System interconnections

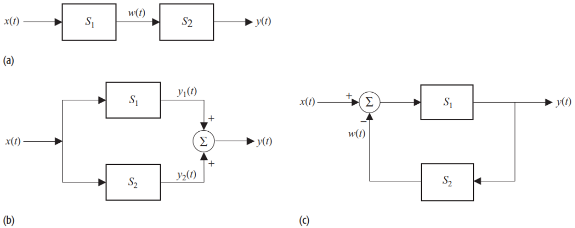

Figure 4: a)Cascaded, b) Parallel and c) Feedback configurations

Cascaded configuration

$$S_1: \quad \frac{dw}{dt} + 2w(t) = x(t) \quad \text{with } w(0) = 0$$
$$S_2: \quad \frac{dy}{dt} + 3y(t) = w(t) \quad \text{with } y(0) = 0 \tag{5}$$

Cascaded configuration

Differentiating both sides of the differential equation modeling system S2 with respect to t yields

$$S_2: \quad \frac{d^2y}{dt^2} + 3\frac{dy(t)}{dt} = \frac{dw(t)}{dt} \tag{6}$$

Multiplying the differential equation modeling system S_2 by 2 and adding the result to the above equation yields

$$\frac{d^2y}{dt^2} + 5\frac{dy(t)}{dt} + 5y(t) = \frac{dw(t)}{dt} + 2w(t) \tag{7}$$

But right side of the above equation is $x(t)$. So we have

$$\frac{d^2y}{dt^2} + 5\frac{dy(t)}{dt} + 5y(t) = x(t) \tag{8}$$

Parallel configuration

The parallel configuration is shown in Fig above (b), where a single input is applied simultaneously to two systems S_1 and S_2. The overall output response is obtained by adding the outputs of the individual systems. In other words, if

$$\begin{aligned} S_1 :& x(t) \rightarrow y_1(t) \text{ and } S_2 : x(t) \rightarrow y_2(t) \\ & \text{then} \\ S_{parallel} :& x(t) \rightarrow y_1(t) + y_2(t) \end{aligned} \tag{9}$$

As for the series configuration, the system formed by a parallel combination of two linear systems is also linear. Similarly, if the two systems S_1 and S_2 are time-invariant, then the overall parallel system is also time-invariant.

Parallel configuration

$$S_1 : y_1(t) = x(t) + \frac{dx(t)}{dt} \text{ and } S_2 : y_2(t) = x(t) + 3\frac{dx(t)}{dt} + 5\frac{d^2x}{dt^2}$$

$$S_{parallel} : y_1(t) + y_2(t) = 2x(t) + 4\frac{dx}{dt} + 5\frac{d^2x}{dt^2} \tag{10}$$

but $y_1(t) + y_2(t) = y(t)$

Feedback configuration

The feedback configuration is shown above (c), where the output of system S_1 is fed back, processed by system S_2, and then subtracted from the input signal Such systems are difficult to analyze in the time domain and will be considered in Chapter 6 after the introduction of the Laplace transform.