Formulas

Properies of δ

$$\int f(\tau)\delta(t-\tau)d\tau = \int f(t-\tau)\delta(\tau)d\tau = f(t)$$

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)\,dx = f(a).$$

If there are finite bounds and a is outside from bounds integral is zero.

Appendix A Mathematical preliminaries

A.1 Trigonometric identities

$$e^{\pm jt} = \cos t \pm j \sin t$$

$$\cos t = \frac{1}{2}[e^{jt} + e^{-jt}]$$

$$\sin t = \frac{1}{2j}[e^{jt} - e^{-jt}]$$

$$\cos\left(t \pm \frac{\pi}{2}\right) = \mp \sin t$$

$$\sin\left(t \pm \frac{\pi}{2}\right) = \pm \cos t$$

$$\sin 2t = 2 \sin t \cos t$$

$$\cos^2 t + \sin^2 t = 1$$

$$\cos^2 t - \sin^2 t = \cos 2t$$

$$\cos^2 t = \frac{1}{2}(1 + \cos 2t)$$

$$\sin^2 t = \frac{1}{2}(1 - \cos 2t)$$

$$\cos^3 t = \frac{1}{4}(3 \cos t + \cos 3t)$$

$$\sin^3 t = \frac{1}{4}(3 \sin t - \sin 3t)$$

$$\cos(t \pm \theta) = \cos t \cos\theta \mp \sin t \sin\theta$$

$$\sin(t \pm \theta) = \sin t \cos\theta \pm \cos t \sin\theta$$

$$\tan(t \pm \theta) = \frac{\tan t \pm \tan\theta}{1 \mp \tan t \tan\theta}$$

$$\sin t \sin\theta = \frac{1}{2}[\cos(t - \theta) - \cos(t + \theta)]$$

$$\cos t \cos\theta = \frac{1}{2}[\cos(t + \theta) + \cos(t - \theta)]$$

$$\sin t \cos\theta = \frac{1}{2}\left[\sin(t+\theta) + \sin(t-\theta)\right]$$

$$a\cos t + b\sin t = C\cos(t+\theta),\ C = \sqrt{a^2+b^2},\ \theta = \tan^{-1}(-b/a)$$

$$a\cos(mt) + b\sin(mt) = \sqrt{a^2+b^2}\cos(mt-\theta),\ \theta = \tan^{-1}\frac{b}{a}$$

$$a\cos(mt) + b\sin(mt) = \sqrt{a^2+b^2}\sin(mt+\phi),\ \phi = \tan^{-1}\frac{a}{b}$$

A.2 Power series

$$\ln(1+t) = t - \frac{t^2}{2} + \frac{t^3}{3} - \frac{t^4}{4} + \cdots$$

$$e^t = 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots$$

$$\sin t = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \cdots$$

$$\cos t = 1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} + \cdots$$

$$\tan t = t + \frac{t^3}{3} + \frac{2t^5}{15} + \frac{17t^7}{315} + \cdots$$

$$\sin^{-1} t = t + \frac{1}{2}\frac{t^3}{3} + \frac{1.3}{2.4}\frac{t^5}{5} + \cdots$$

A.3 Series summation

Arithmetic series

$$\sum_{n=1}^{n}[a + (n-1)d] = \frac{N}{2}[2a + (N-1)d]$$

$$\sum_{n=1}^{n} n = 1 + 2 + \cdots + N = \frac{N(N+1)}{2}$$

Geometric series

$$\sum_{n=0}^{N} ar^n = \frac{a(1-r^{N+1})}{1-r}$$

$$\sum_{n=0}^{N-1} \exp\left[j\frac{2\pi kn}{N}\right] = \begin{cases} 0 & 1 \le k \le (N-1) \\ N & k = 0, \end{cases}$$

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r},\ |r| < 1$$

$$\sum_{n=0}^{\infty} nr^n = \frac{r}{(1-r)^2},\ |r| < 1$$

The geometric progression (GP) series sum of the form

$$S = \sum_{n=0}^{N} ar^n = a + ar + ar^2 + \cdots + ar^N$$

is used frequently in this text while dealing with the discrete-time signals. Note that the factor r can be real, imaginary, or complex.

A.4 Limits and differential calculus

$$\lim_{t\to\infty} t^{-\alpha} \ln t = 0, \quad \mathrm{Re}(\alpha) > 0$$

$$\lim_{t\to 0} t^{\alpha} \ln t = 0, \quad \mathrm{Re}(\alpha) > 0$$

L'Hôpital's Rule:

If $\lim_{t\to a} x(t) = \lim_{t\to a} y(t) = 0$ or $\lim_{t\to a} x(t) = \lim_{t\to a} y(t) = \infty$, and $\lim_{t\to a} \frac{x'(t)}{y'(t)}$ has a finite value, then $\lim_{t\to a} \frac{x(t)}{y(t)} = \lim_{t\to a} \frac{x'(t)}{y'(t)}$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{1}{g(t)}\right\} = -\frac{1}{g^2(t)}\frac{\mathrm{d}g(t)}{\mathrm{d}t}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{\frac{h(t)}{g(t)}\right\} = \frac{1}{g^2(t)}\left[g(t)\frac{\mathrm{d}h(t)}{\mathrm{d}t} - h(t)\frac{\mathrm{d}g(t)}{\mathrm{d}t}\right]$$

A.5 Indefinite integrals

$$\int u\,\mathrm{d}v = uv - \int v\,\mathrm{d}u$$

$$\int f(t)g(t)\,\mathrm{d}t = f(t)\int g(t)\mathrm{d}t - \int\left[\frac{\mathrm{d}f}{\mathrm{d}t}\int g(t)\mathrm{d}t\right]\mathrm{d}t$$

$$\int \cos at\,\mathrm{d}t = \frac{1}{a}\sin at + C,\ a \neq 0$$

$$\int \sin at\,\mathrm{d}t = -\frac{1}{a}\cos at + C,\ a \neq 0$$

$$\int \cos^2 at\,\mathrm{d}t = \frac{t}{2} + \frac{\sin 2at}{4a} + C,\ a \neq 0$$

$$\int \sin^2 at\,\mathrm{d}t = \frac{t}{2} - \frac{\sin 2at}{4a} + C,\ a \neq 0$$

$$\int t\cos at\,\mathrm{d}t = \frac{1}{a^2}(\cos at + at\sin at) + C,\ a \neq 0$$

$$\int t\sin at\,\mathrm{d}t = \frac{1}{a^2}(\sin at - at\cos at) + C,\ a \neq 0$$

$$\int t^2 \cos at \, dt = \frac{1}{a^3}(2at \cos at - 2\sin at + a^2t^2 \sin at) + C, a \neq 0$$

$$\int t^2 \sin at \, dt = \frac{1}{a^3}(2at \sin at - 2\cos at - a^2t^2 \cos at) + C, a \neq 0$$

$$\int \cos at \cos bt \, dt = \frac{\sin(a-b)t}{2(a-b)} + \frac{\sin(a+b)t}{2(a+b)} + C, a^2 \neq b^2$$

$$\int \sin at \sin bt \, dt = \frac{\sin(a-b)t}{2(a-b)} - \frac{\sin(a+b)t}{2(a+b)} + C, a^2 \neq b^2$$

$$\int \sin at \cos bt \, dt = -\left[\frac{\cos(a-b)t}{2(a-b)} + \frac{\cos(a+b)t}{2(a+b)}\right] + C, a^2 \neq b^2$$

$$\int \sin^{-1} at \, dt = t \sin^{-1} at + \frac{1}{a}\sqrt{1 - a^2t^2} + C, a \neq 0$$

$$\int \cos^{-1} at \, dt = t \cos^{-1} at - \frac{1}{a}\sqrt{1 - a^2t^2} + C, a \neq 0$$

$$\int e^{at} dt = \frac{1}{a} e^{at} + C, a \neq 0$$

$$\int b^{at} dt = \frac{b^{at}}{a \ln b} + C, a \neq 0, b > 0, b \neq 1$$

$$\int t e^{at} dt = \frac{e^{at}}{a^2}(at - 1) + C, a \neq 0$$

$$\int t^2 e^{at} dt = \frac{e^{at}}{a^3}(a^2t^2 - 2at + 2) + C, a \neq 0$$

$$\int t^n e^{at} dt = \frac{1}{a} t^n e^{at} - \frac{n}{a} \int t^{n-1} e^{at} dt, a \neq 0$$

$$\int t^n b^{at} dt = \frac{t^n b^{at}}{a \ln b} - \frac{n}{a \ln b} \int t^{n-1} b^{at} dt, a \neq 0, \ b > 0, b \neq 1$$

$$\int e^{at} \sin bt \, dt = \frac{e^{at}}{a^2 + b^2}(a \sin bt - b \cos bt) + C$$

$$\int e^{at} \cos bt \, dt = \frac{e^{at}}{a^2 + b^2}(a \cos bt + b \sin bt) + C$$

$$\int t^n \ln at \, dt = \frac{t^{n+1}}{n+1} \ln at - \frac{t^{n+1}}{(n+1)^2} + C, n \neq -1$$

$$\int \frac{1}{t^2 + a^2} dt = \frac{1}{a} \tan^{-1}\left(\frac{t}{a}\right) + C, a \neq 0$$

$$\int \frac{t}{t^2 + a^2} dt = \frac{1}{2} \ln(t^2 + a^2) + C$$